GREENLAWNS SCHOOL, WORLI

Final Examination 2018

MATHEMATICS

STD: IX — Marks: 80

Date: 23/02/2018 — Time: 2½hrs

Question 1

a. The diagonals of a rhombus are 12 cm and 16 cm. Find (i) the length of its one side (ii) its perimeter and area **[3]**

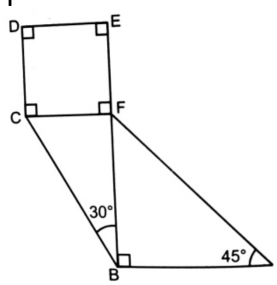
b. Find the equation of line whose x-intercept = – 8 & y-intercept = – 4. **[3]**

c. Find the solution of the following equations graphically: $2x - 3y = -6$ and $x - y/2 = 1$ **[4]**

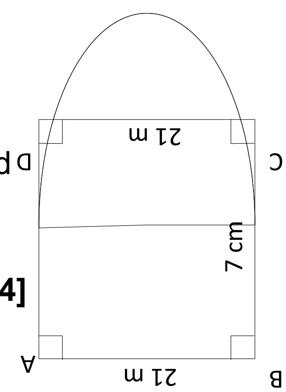
Question 2

a. Find a point on x-axis which is equivalent from points (7, 6) & (–3, 4). **[3]**

b. CDEF is a square of area 4 cm^2. EF is produced to B and angle CBF= 30 .AB is perpendicular to BF and angle A = 45 .Find BF and AF **[3]**



c. The given figure shows cross section of a water channel consisting of rectangle & a semi-circle. Assuming that the channel is always full, find the volume of water discharged through it in one minute if water is flowing at the rate of 20 cm/sec. Give your answer in cubic metre correct up to one place of decimal. **[4]**



Question 3

a. The points (k, 3), (2, –4) & (–k + 1, –2) are collinear. Find K. **[3]**

b. The marks of 20 students in a test were as follows:
2, 6, 8, 9, 10, 11, 14, 15, 15, 15, 16, 16, 11, 12, 13, 13, 14, 18, 19 & 20.
Calculate: (i) the mean, (ii) the median, (iii) the mode **[3]**

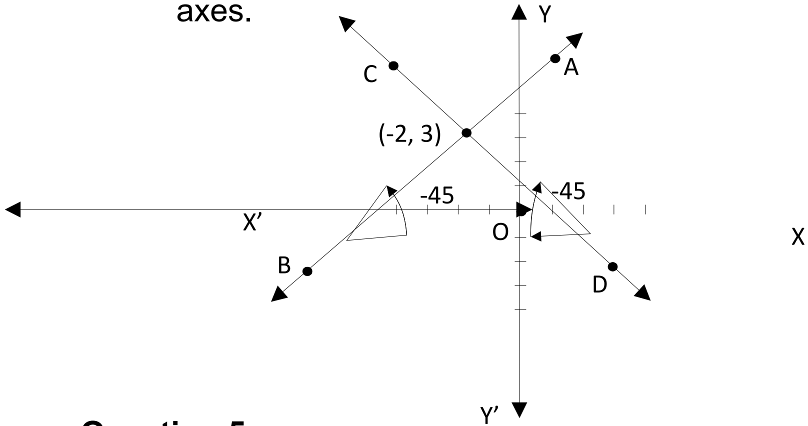
c. Find the area of trapezium whose parallel sides are 15 cm and 23 cm ; whereas the non parallel sides are 10 cm and 8 cm **[4]**

Question 4

a. Evaluate: $14\sin 30° + 6\cos 60° – 5\tan 45°$. **[3]**

b. If $4\cos^2 A – 3 = 0$, show that: $\cos 3A = 4\cos^3 A – 3\cos A$. **[3]**

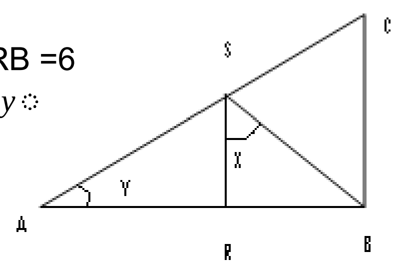
c. Find the equation of lines which pass through point (–2, 3) & are equally inclined to co-ordinate axes. **[4]**



Question 5

a. length and breadth of a playground are 24m by 20 m. find the cost of covering it with gravel 1.5 m deep, if the gravel costs Rs 15 per m^3. **[2]**

b. In the adjoining figure AB = 18, SR = 5 and RB =6 **[4]**
Find the i. $tan x$ ii . $sin y$. iii $cos y$



c. For the following frequency distribution draw a histogram. Hence calculate the mode. **[4]**

Class	5-10	10-15	15-20	20-25	25-30
Frequency	7	18	10	8	5

Question 6

a. In 10 numbers, arranged in increasing order the 7th number is increased by 8, how much will the median and mean will change. **[2]**

b. Construct the frequency polygon of following data without histogram **[4]**

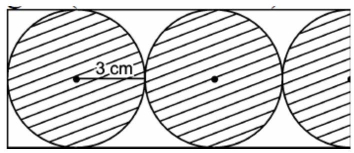
C.I	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
f	5	8	15	21	25	15	8	6

c. Prove that the points A(–5, 4); B(–1, –2) & (5, 2) are vertices of an isosceles right angled triangle. Find the co-ordinates of D, so that ABCD is a square. **[4]**

Question 7

a. If the lines $y = 3x + 7$ & $2y + PX = 3$ are perpendicular to each other. Find value of p. **[3]**

b. In the given figure, find the area of the unshaded portion within the rectangle. ($\pi = 3.14$) **[3]**



c. A (5, 4), B (–3, –2) & C(1, –8) are the vertices of $\triangle ABC$. Find:

i) Slope of altitude of AB.

ii) Slope of median AD.

iii) Slope of line parallel to AC. **[4]**

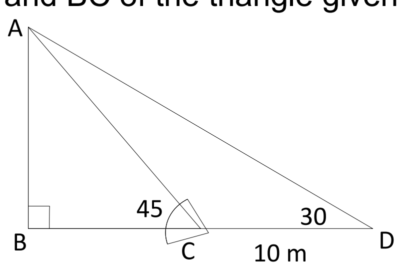
Question 8

a. Find the equation of line whose y intercept = –1 & inclination = 45°. **[2]**

b. Evaluate: $\dfrac{\sin 35^\circ \cos 55^\circ + \cos 35^\circ \sin 55^\circ}{\operatorname{cosec}^2 10^\circ - \tan^2 80^\circ}$. **[3]**

c. Evaluate: $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$. **[2]**

d. Find the length of AB and BC of the triangle given below **[3]**



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Answer key

Question 1

a. The diagonals of a rhombus are 12 cm and 16 cm. Find (i) the length of its one side (ii) its perimeter and area **[3]**

b. Find the equation of line whose x-intercept = – 8 & y-intercept = – 4. **[3]**

Solution:

x-intercept = –8

$\therefore$ corresponding point on x-axis = (–8, 0)

y-intercept = –4

$\therefore$ corresponding point on y-axis = (0, –4)

For equation of line:

$(-8, 0) = (x_1, y_1)$; $(0, -4) = (x_2, y_2)$

Using two point form:

$$\frac{y - y_1}{y_1 - y_2} = \frac{x - x_1}{x_1 - x_2}$$

$$\therefore \frac{y - 0}{0 - (-4)} = \frac{x - (-8)}{-8 - 0}$$

$$\therefore \frac{y}{4} = \frac{x + 8}{-8}$$

$\therefore -8y = 4x + 32$

$\therefore 4x + 8y + 32 = 0$

$\therefore x + 2y + 8 = 0$ is the equation

c. Find the solution of the following equations graphically: 2x – 3y = - 6 and x-y/2 =1 **[4]**

Question 2

a. Find a point on x-axis which is equivalent from points (7, 6) & (–3, 4). **[3]**

Solution:

Let P(x, 0) be the point on x-axis

A ≡ (7, 6) & B ≡ (–3, 4)

According to given condition,

PA = PB

By distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\therefore \sqrt{(x-7)^2 + (0-6)^2} = \sqrt{[x-(-3)]^2 + [0-4]^2}$

Squaring both sides, we get,

$x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$

Using, $(a-b)^2 = a^2 - 2ab + b^2$ &

$(a+b)^2 = a^2 + 2ab + b^2$

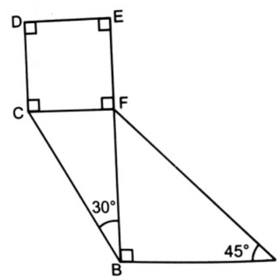
$\therefore -14x - 6x = 25 - 85$

$-20x = -60$

$\therefore x = 3$

$\therefore$ P(3, 0) is required point

b. CDEF is a square of area 4 cm^2. EF is produced to B and angle CBF= 30 .AB is perpendicular to BF and angle A = 45 .Find BF and AF **[3]**



Ans CF = √4 = 2

Tan30 = CF/BF

1/√3 = 2/BF

BF = 2√3 = 2 x 1.732 = 3.464 cm

Δ FBA is isco triangle

So BF= AF = 3.464 cm

c. The given figure shows cross section of a water channel consisting of rectangle & a semi-circle. Assuming that the channel is always full, find the volume of water discharged through it in one minute if water is flowing at the rate of 20 cm/sec. Give your answer in cubic metre correct up to one place of decimal. **[4]**

Solution:

For rectangle ABCD,

AB = DC = 21 cm

AD = BC = 7 cm

For semi-circle,

Diameter = DC = 21 cm

Radius (r) = $\frac{21}{2}$ cm

Rate i.e. speed of water = 20 m/sec

Time = one minute = 60 sec

Distance covered by water = speed × time

= 20 × 60

= <u>1200 cm</u>

Length of channel = Distance covered by water

$= 1200$ cm

Volume of water $=$ Area of cross section $\times$ Length of channel

$$= \left[(l \times b) + \frac{\pi r^2}{2}\right] \times 1200$$

$$= \left[(21 \times 7) + \frac{22 \times 21 \times 21}{7 \times 2 \times 2 \times 2}\right] \times 1200$$

$$= \left[147 + \frac{693}{4}\right] \times 1200$$

$$= \frac{1281}{4} \times 1200 = 1281 \times 300 \text{ cm}^3$$

$$\text{Volume of water} = \frac{1281 \times 300}{100 \times 100 \times 100} \text{ m}^3 = \frac{3843}{10000} = \underline{0.4 \text{ m}^3}$$

Question 3

a. The points (k, 3), (2, –4) & (–k + 1, –2) are collinear. Find K**.** **[3]**

Solution:

Let A = (k, 3); B = (2, –4) & C = (–k + 1, –2)

Slope of AB = Slope of Bc

∵ A, B, C are collinear… given.

For slope of AB & Slope of BC

$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$ $m_2 = \frac{y_2 - y_1}{x_2 - x_1}$

$m_1 = \frac{-4 - 3}{2 - k}$ $m_2 = \frac{-2 - (-4)}{-k + 1 - 2}$

$\because \Rightarrow m_1 = m_2$

$$\therefore \frac{-7}{2 - k} = \frac{2}{-k - 1}$$

$\therefore 7k + 7 = 4 - 2k$

$\therefore 9k = -3$

$$k = \frac{-1}{3}$$

b. The marks of 20 students in a test were as follows:

2, 6, 8, 9, 10, 11, 14, 15, 15, 15, 16, 16, 11, 12, 13, 13, 14, 18, 19 & 20.

Calculate: (i) the mean, (ii) the median, (iii) the mode **[3]**

Solution:

i) $\sum x$ = 2 + 6 + 8 + 9 + 10 + 11 + 11 + 12 + 13 + 13 + 14 + 14 + 15 + 15 + 15 + 16 + 16 + 18 + 19 + 20.

$\sum x$ = 257

n = 20

Mean = $\frac{\sum x}{n}$

= $\frac{257}{20}$

Mean = 12.85

ii) N = 20 (even)

Median = $\frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$

= $\frac{\left(\frac{20}{2}\right)^{th} \text{term} + \left(\frac{20}{2}+1\right)^{th} \text{term}}{2}$

= $\frac{10^{th} \text{term} + 11^{th} \text{term}}{2}$

= $\frac{13+14}{2}$

= $\frac{27}{2}$

Median = 13.5

iii) No. with highest frequency = 15

$\therefore$ Mode = 15

c. Find the area of trapezium whose parallel sides are 15 cm and 23 cm ; whereas the non parallel sides are 10 cm and 8 cm **[4]**

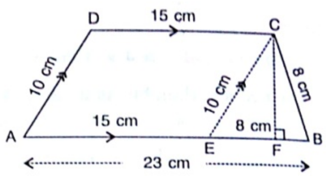
Ans.

DC = 15 cm, AB = 23 cm;

AD = 10 cm and BC = 8 cm.

Draw CE parallel to DA which meets AB at point E.

Since, opposite sides of the quadrilateral AECD are parallel, it is parallelogram



Also, draw CF perpendicular to EB

In triangle EBC,

Let a = EB = AB – AE = AB – DC = (23 – 15) cm = 8 cm

b = CE = DA = 10 cm and c = BC = 8 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{8+10+8}{2} \text{ cm} = 13 \text{ cm}$$

$$\text{Area of } \Delta \text{ EBC} = \sqrt{13(13-8)(13-10)(13-8)} \text{ cm}^2$$

$$= \sqrt{13\times5\times3\times5} \text{ cm}^2 = 5\sqrt{39} \text{ cm}^2$$

If CF is taken height corresponding to the base BE,

$$\text{Area of } \Delta \text{ EBC} = \frac{1}{2} \times \text{EB} \times \text{CF cm}^2$$

$$\therefore \quad \frac{1}{2} \times 8 \times \text{CF} = 5\sqrt{39} \quad \Rightarrow \text{CF} = \frac{5\sqrt{39}}{4} \text{ cm}$$

Clearly, distance between the parallel sides AB and DC is the length of CF.

$\therefore$ **Area of given trapezium** $= \frac{1}{2}$(sum of parallel sides) × distance between them

$$= \frac{1}{2}(15 + 23) \times \text{CF}$$

$$= \frac{1}{2} \times 38 \times \frac{5\sqrt{39}}{4} \text{ cm}^2 = 148{\cdot}32 \text{ cm}^2$$ **Ans**

Question 4

a. Evaluate: 14sin 30° + 6cos 60° – 5tan 45°. [3]

Solution:

14 sin 30° + 6 cos 60° – 5 tan 45°

$= 14 \times \frac{1}{2} + 6 \times \frac{1}{2} - 5 \times 1$

= 7 + 3 – 5

= 10 – 5

= 5

b. If 4 $\cos^2$ A – 3 = 0, show that: cos 3A = 4 $\cos^3$ A – 3 cos A. [3]

Solution:

$4\cos^2 A - 3 = 0$

$\therefore 4\cos^2 A = 3$

$\cos^2 A = \frac{3}{4}$ … taking square root

$\therefore \cos A = \frac{\sqrt{3}}{2}$

But $\cos 30^\circ = \dfrac{\sqrt{3}}{2}$

$\therefore \cos A = \cos 30^\circ$

$\therefore A = 30^\circ$

Now, L.H.S. $= \cos 3A$

$= \cos 3(30)$

$= \cos 90^\circ$

L.H.S. $= 0$ … (1)

R.H.S. $= 4\cos^3 A - 3\cos A$

$= 4\cos^3 30^\circ - 3\cos 30^\circ$

$= 4\left(\dfrac{\sqrt{3}}{2}\right)^3 - 3\left(\dfrac{\sqrt{3}}{2}\right)$

$= 4 \times \dfrac{3\sqrt{3}}{8} - \dfrac{3\sqrt{3}}{2}$

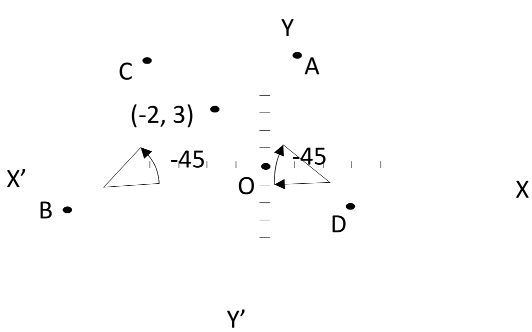
$= \dfrac{3\sqrt{3}}{2} - \dfrac{3\sqrt{3}}{2}$

R.H.S. $= 0$ … (2)

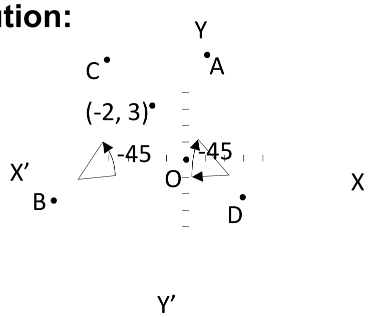
from (1) & (2)

$\therefore$ L.H.S. = R.H.S. … hence proved.

c. Find the equation of lines which pass through point (–2, 3) & are equally inclined to co-ordinate axes. [4]



Solution:



Let AB & CD be two lines equally inclined to co-ordinate axes.

For line AB:

Inclination $(\theta) = 45$

$m = \tan\theta$

$m = \tan 45$
$m = 1$

For equation of line AB:

$m = 1$, $(x_1, y_1) = (-2, 3)$

Using slope point form,

$y - y_1 = m(x - x_1)$
$y - 3 = 1[x - (-2)]$
$y - 3 = x + 2$
$\therefore \underline{y = x + 5}$

For line CD:

Inclination $(\theta) = -45°$
$m = \tan \theta$
$= \tan(-45)$
$m = \underline{-1}$

For equation of line CD:

$m = -1$, $(x_1, y_1) = (-2, 3)$

Using slope point form,

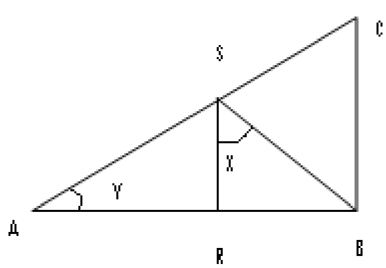
$y - y_1 = m(x - x_1)$
$y - 3 = -1[x - (-2)]$
$y - 3 = -x - 2$
$\therefore x + y = 1$
$\underline{y = -x + 1}$

Question 5

a. length and breadth of a playground are 24m by 20 m. find the cost of covering it with gravel 1.5 m deep , if the gravel costs Rs 15 per m^3 . **[2]**

Ans. Cost = volume X rate
= (24 x 20 x 1.5) x 15
= Rs.10800

b. In the adjoining figure AB = 18, SR = 5 and RB =6 **[4]**
Find the i. *tanx* ii . *siny* . iii *cosy*



Solution :

(i) In right-angled triangle SRB.

$$\tan x° = \frac{\text{perpendicular}}{\text{base}} = \frac{RB}{RS} = \frac{6}{5}$$

(ii) In right-angled triangle ARS,

$AR = 18 - 6 = 12$

$AS^2 = AR^2 + SR^2 = 12^2 + 5^2 = 144 + 25 = 1$

$\therefore AS = 13.$

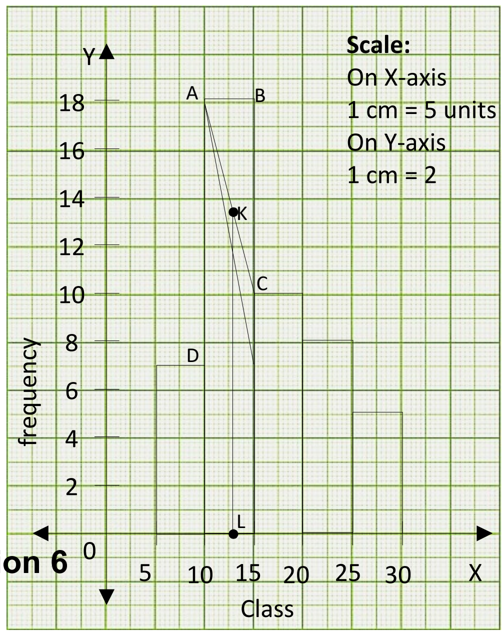
$$\therefore \sin y° = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{SR}{AS} = \frac{5}{13}$$

(iii) $$\cos y° = \frac{\text{base}}{\text{hypotenuse}} = \frac{AR}{AS} = \frac{12}{13}$$

c. **For the following frequency distribution draw a histogram. Hence calculate the mode.** **[4]**

Class	**5-10**	**10-15**	**15-20**	**20-25**	**25-30**
Frequency	**7**	**18**	**10**	**8**	**5**

Solution:



From Graph:
Join AC & BD
Draw KL $\perp^r$ X-axis
Mode = <u>13</u>

Question 6

a. In 10 numbers, arranged in increasing order the 7^{th} number is increased by 8, how much will the median and mean will change. **[2]**

Ans. Median will not change but mean will increase by 0.8

b. Construct the frequency polygon of following data without histogram **[4]**

C.I	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
f	5	8	15	21	25	15	8	6

c. Prove that the points A(–5, 4); B(–1, –2) & (5, 2) are vertices of an isosceles right angled triangle. Find the co-ordinates of D, so that ABCD is a square. **[4]**

Solution:

By distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

For AB:

A(–5, 4) = (x_1, y_1); B(–1, –2) = (x_2, y_2)

AB = $\sqrt{[-1-(-5)]^2 + [-2-4]^2}$

AB = $\sqrt{(4)^2 + (-6)^2}$

AB = $\sqrt{16 + 36}$

AB = $\sqrt{52}$

AB = $2\sqrt{13}$ units … (i)

For BC:

A(–1, –2) = (x_1, y_1); C(5, 2) = (x_2, y_2)

BC = $\sqrt{[5 - (-1)^2] + [2 - (-2)]^2}$

BC = $\sqrt{(6)^2 + (4)^2}$

BC = $\sqrt{36 + 16}$

BC = $\sqrt{52}$

BC = $2\sqrt{13}$ units … (ii)

$\therefore$ AB = BC … from (i) & (ii)

$\therefore \Delta$ABC is an Isosceles Δ^{le}

For AC:

A(–5, 4) = (x_1, y_1) & C(5, 2) = (x_2, y_2)

AC = $\sqrt{[5 - (-5)]^2 + [2 - 4]^2}$

AC = $\sqrt{(10)^2 + (2)^2}$

AC = $\sqrt{104}$

AC = $2\sqrt{26}$ units… (iii)

$(AC)^2 = \left(2\sqrt{26}\right)^2 = 4 \times 26 = 104$

$(AB)^2 + (BC)^2 = \left(2\sqrt{3}\right)^2 + \left(2\sqrt{13}\right)^2$

$= 4 \times 13 + 4 \times 13$

$= 52 + 52$

$= 104$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

By converse of Pythagoras theorem, ΔABC is isosceles right $\angle^{led}$, Δ^{le}.

For D:

Such that ☐ ABCD is square

Let diagonals AC & BD intersect at point 'O'.

$\therefore$ O is mid-point of AC & BD.

A(–5, 4) & C(5, 2)

By mid-point formula,

$$O(x, y) = \left\{\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right\}$$

$$= \left\{\frac{-5+5}{2}, \frac{4+2}{2}\right\}$$

O(x, y) = (0, 3)

B(–1, –2) & D(x_2, y_2)

$$O(x, y) = \left\{\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right\}$$

$$O(0, 3) = \left\{\frac{-1+x_2}{2}, \frac{-2+y_2}{2}\right\}$$

$$\therefore 0 = \frac{-1+x_2}{2} \quad \& \quad 3 = \frac{-2+y_2}{2}$$

$$\therefore -1 + x_2 = 0 \quad\quad 6 = -2 + y_2$$

$$x_2 = 1 \quad\quad \therefore y_2 = 8$$

$\therefore$ D(x_2, y_2) = (1, 8)

Question 7

a. If the lines y = 3x + 7 & 2y + px = 3 are perpendicular to each other. Find value of p. **[3]**

Solution:

Equation of first line ⇒ y = 3x + 7

$\therefore$ Its slope (m_1) = 3

Equation of other line ⇒ 2y + px = 3

writing as y = mx + c

$\therefore$ 2y = –px + 3

$$y = \frac{-p}{2}x + \frac{3}{2}$$

$$\text{Its slope } (m_2) = \frac{-p}{2}$$

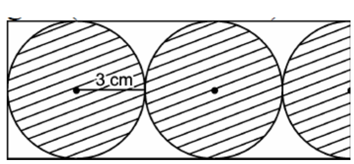
Now, $m_1 \times m_2 = -1$ [$\because \perp^r$ lines]

$$3 \times \frac{-p}{2} = -1$$

$$\frac{3p}{2} = 1$$

$$\therefore p = \frac{2}{3}$$

b. In the given figure, find the area of the unshaded portion within the rectangle. (Take π = 3.14) [3]



Ans. Area = 15 x6 – 2.5 π r^2

$= 90 - 2.5 \times 3.14 \times 3 \times 3$
$= 90 - 70.65$
$= 19.35$ Sq cm.

c. A (5, 4), B(–3, –2) & C(1, –8) are the vertices of ΔABC. Find:
i) Slope of altitude of AB.
ii) Slope of median AD.
iii) Slope of line parallel to AC**.**

Solution:

i) Let m_1 be slope of AB & m_2 be slope of its altitude.

For m_1:

$A(5, 4) = (x_1, y_1)$; $B(-3, -2) = (x_2, y_2)$

$$m1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m1 = \frac{-2 - 4}{-3 - 5} = \frac{-6}{-8} = \frac{3}{4}$$

Now, m_1& m_2 = –1 [$\because \perp^r$ lines]

$$\frac{3}{4} \times m_2 = -1$$

$$m_2 = \frac{-4}{3}$$

Slope of altitude of AB = $\frac{-4}{3}$

ii) AD is median … given

$\therefore$ D is mid-point of BC

For D:

$B(-3, -2) = (x_1, y_1)$
$C(1, -8) = (x_2, y_2)$

By mid-point formula,

$$D(x, y) = \left\{\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right\}$$

$$D(x, y) = \left\{\frac{-3 + 1}{2}, \frac{-2 - 8}{2}\right\}$$

$D(x, y) = (-1, -5)$

Slope of AD:

$A(5, 4) = (x_1, y_1)$
$D(-1,-5) = (x_2, y_2)$

$$\text{Slope } (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-5-4}{-1-5} = \frac{-9}{-6} = \frac{3}{2}$$

iii) Let m_1 be slope of AC & m_2 be slope of line parallel to AC.

For m_1:

$A(5, 4) = (x_1, y_1)$

$D(1,-8) = (x_2, y_2)$

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_1 = \frac{-8-4}{1-5} = \frac{-12}{-4} = 3$$

Now, $m_1 = m_2$ [$\because$ || lines]

$\therefore m_2 = 3$

$\therefore$ Slope of line parallel to AC = 3

Question 8

a. Find the equation of line whose y intercept = –1 & inclination = 45°. [2]

Solution:

y intercept (c) = –1

Inclination (θ) = 45°

Slope (m) = $\tan\theta$

= $\tan 45°$

m = 1

Using slope intercept form:

y = mx + c

y = 1(x) + (–1)

y = x – 1

b. Evaluate: $\dfrac{\sin 35°\cos 55° + \cos 35°\sin 55°}{\operatorname{cosec}^2 10° - \tan^2 80°}$. [3]

Solution:

$$\frac{\sin 35°\cos 55° + \cos 35°\sin 55°}{\operatorname{cosec}^2 10° - \tan^2 80°}$$

$$= \frac{\sin 35 \cdot \sin(90-55)° + \cos 35 \cdot \cos(90-55)°}{\operatorname{cosec}^2 10 - \cot^2(90-80)°}$$

$$= \frac{\sin 35° \cdot \sin 35° + \cos 35° \cdot \cos 35°}{\operatorname{cosec}^2 10 - \cot^2 10}$$

$$= \frac{\sin^2 35° + \cos^2 35°}{\operatorname{cosec}^2 10 - \cot^2 10}$$

$= \frac{1}{1}$ … $\sin^2\theta + \cos^2\theta = 1$, $\text{cosec}^2\theta - \cot^2\theta = 1$

= 1

c. Evaluate: 3 cos 80° cosec 10° + 2 cos 59° cosec 31°. [2]

Solution:

3 cos 80°· cosec 10° + 2 cos 59°· cosec 31°

= 3 sin (90 – 80)°· cosec 10° + 2 sin(90 – 59)° cosec 31°

= 3 sin 10°· cosec 10° + 2 sin 31°· cosec 31°

= 3 × 1 + 2 × 1 … sin θ· cosec θ = 1

= 3 + 2

= 5

d. Find the length of AB and BC of the triangle given below [3]