

GREENLAWNS SCHOOL, WORLI

First Term Examination - 2017

MATHEMATICS

STD: IX

Marks: 80

Date: /02/2017

Time: 2½hrs

Question 1

- a. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$ [3]
- b. A boy scored following marks in various class tests during a term ; each test being marked out of 20. are 15,17,16,7,10,12,14,16,19,12,16
 1-What are his modal marks ?
 2-What are his median marks ?
 3-What are his mean marks ? [3]
- c. Draw a histogram for the following frequency distribution and hence find the mode. [4]

| | | | | | |
|-----------|-------|-------|-------|-------|-------|
| CI | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 |
| Frequency | 24 | 16 | 9 | 15 | 20 |

Question 2

- a. Marks obtained (in mathematics) by students are given below.
 60,67,52,76,50,51,74,45,46
 i. Find the arithmetic mean.
 ii. If marks of each student be increased by 4; what will be the new value of arithmetic mean. [3]
- b. Evaluate: $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$. [3]
- c. From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a cylindrical cavity of diameter 14 cm & depth 24 cm is drilled out. Find:
 i) Surface area of remaining solid
 ii) Volume of remaining solid
 iii) Weight of material drilled out if it weighs 7 gm per cm^3 [4]

Question 3

- a. From the following data ,
 25,10,40,88,45,60,77,36,18,95,56,65,7,0,38 and 83.find :
 i Median
 ii Upper quartile
 iii Inter-quartile range [3]
- b. The volume of a rectangular solid is 3600 cm^3 . If it is 20 cm long and 9 cm high, find its total surface area. [3]

- c. Construct a triangle XYZ with angle Y equals to 75° and $XY = 5.5$ cm and $YZ = 6.5$ cm. and draw in circle in it. [4]

Question 4

- a. The mean of the numbers 6, x, 7, y and 14 is 8. Express y in terms of x. [3]
 b. A rectangular container whose base is square of side 6 cm holds water up to 3 cm from the top. When a solid cube is placed in the water and is completely submerged, the water rises to the top and 17 cm^3 of water overflows. Find the edge of the cube. [3]
 c. The marks of 200 students in a test were recorded as follows:

| Marks % | No. of students |
|---------|-----------------|
| 10-19 | 7 |
| 20-29 | 11 |
| 30-39 | 20 |
| 40-49 | 46 |
| 50-59 | 57 |
| 60-69 | 37 |
| 70-79 | 15 |
| 80-89 | 7 |

Construct the frequency polygon of above frequency distribution. [4]

Question 5

- a. Construct a triangle ABC with angle B equals to 90° and $AC = 6.5$ cm and $BC = 6$ cm. and draw circum circle on it. [3]
 b. $1 - 2[(\tan 35^\circ / \cot 55^\circ)]^2 + [(\cot 55^\circ / \tan 35^\circ)]^2 - 3[(\sec 30^\circ / \operatorname{cosec} 60^\circ)]$ [3]
 c. In each case given below find the value of $\angle A$, where $0 \leq A \leq 90$
 i. $\sin (90 - 3A) \operatorname{cosec} 42^\circ = 1$
 ii. $\cos (90 - A) \sec 77^\circ = 1$ [4]

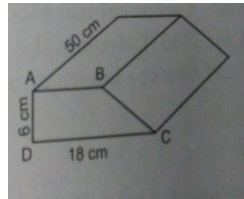
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Question 6

- a. Draw an inscribing circle of a regular hexagon of side 5.8 cm and measure its radius. [3]
 b. If $2 \sin A - 1 = 0$. Show that: $\sin 3A = 3 \sin A - 4 \sin^3 A$. [3]
 c. The height to the nearest cm, of 30 men is given below:
 159,170,174,173,175,160,161,164,163,165,164,171,162,170,177,185,181,180,175,165,
 186,174,168,168,176,176,165,175,167,180 Using class intervals 155-160,160-165, -----
 draw up a grouped frequency distribution table and also find its cumulative frequency [4]

Question 7

- a. Evaluate: $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$. [3]
- b. The marks of 20 students in a test were as follows:
2, 6, 8, 9, 10, 11, 11, 12, 13, 13, 14, 14, 15, 15, 15, 16, 16, 18, 19 & 20.
Calculate: (i) the mean, (ii) the median, (iii) the mode [3]
- c. The diagram represents a solid block of wood of length 50 cm. the cross section is trapezium ABCD with AB = 10 cm, AD = 6cm and CD = 18 cm.



- Find: i. its volume ii total surface area [4]

Question 8

- a. A cylinder with radius 333 units and its volume is 868686 cubic units. Find the height of the cylinder. [3]
- b. Prove that $\frac{\cos A}{1 + \sin A} + \tan A = \sec A$ [3]
- c. Draw a histogram and hence estimate the mode for the following frequency distribution: [4]

| Class | Frequency |
|-------|-----------|
| 0-10 | 2 |
| 10-20 | 8 |
| 20-30 | 10 |
| 30-40 | 5 |
| 40-50 | 4 |
| 50-60 | 3 |

Answer key (std 9 math final 16-17)

Question 1

a. $(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) = 2$

[3]

Solution:

$$\begin{aligned} \text{L.H.S. } & (1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A) \\ & (1 + \frac{\cos A}{\sin A} - \frac{1}{\sin A})(1 + \frac{\sin A}{\cos A} + \frac{1}{\cos A}) \\ & = \frac{\sin A + \cos A - 1}{\sin A} \times \frac{\sin A + \cos A + 1}{\cos A} \\ & = (\sin A + \cos A - 1)(\sin A + \cos A + 1) \sin A \cos A \\ & = (\sin^2 A + \cos^2 A + 2\sin A \cos A - 1) \sin A \cos A \\ & = 1 + 2\sin A \cos A - 1 \sin A \cos A \\ & = 2\sin A \cos A \sin A \cos A \end{aligned}$$

b A boy scored following marks in various class tests during a term ; each test being marked out of 20. are 15,17,16,7,10,12,14,16,19,12,16

- 1-What are his modal marks ?
- 2-What are his median marks ?
- 3-What are his mean marks ?

[3]

Solution-

- 1- Arranging the data in ascending order
7,10,12,12,14,15,16,16,17,19
1-modal marks
16 (maximum occurrence)
- 2- median marks
 $11 + \frac{1}{2} = 6\text{th term} = 15$
- 3- total marks 154 (addition of all)
mean marks
 $\text{Mean} = \frac{\sum fix_i}{\sum fi}$
 $= \frac{154}{11} = 14.$

c. Draw a histogram for the following frequency distribution and hence find the mode.

[4]

| | | | | | |
|-----------|-------|-------|-------|-------|-------|
| CI | 30-39 | 40-49 | 50-59 | 60-69 | 70-79 |
| Frequency | 24 | 16 | 9 | 15 | 20 |

Question 2

a. Marks obtained (in mathematics) by students are given below.
60,67,52,76,50,51,74,45,46

- i. Find the arithmetic mean.
- ii. If marks of each student be increased by 4; what will be the new value of arithmetic mean. [3]

Ans.

- i.
$$\begin{aligned} x' &= x_1 + x_2 + x_3 + \dots + x_n / n \\ &= 60 + 67 + 52 + 76 + 50 + 51 + 74 + 45 + 56 / 9 \\ &= 531 / 9 \\ &= 59 \end{aligned}$$
- ii. If marks of each student be increased by 4; the new value of arithmetic mean will be $= 59 + 4 = 63$

- b. Evaluate: $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$. [3]

Solution:

$$\begin{aligned} & \frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ} \\ &= \frac{\cos 75^\circ}{\cos(90 - 15)^\circ} + \frac{\sin 12^\circ}{\sin(90 - 78)^\circ} - \frac{\cos 18^\circ}{\cos(90 - 72)^\circ} \\ &= \frac{\cos 75^\circ}{\cos 75^\circ} + \frac{\sin 12^\circ}{\sin 12^\circ} - \frac{\cos 18^\circ}{\cos 18^\circ} \\ &= 1 + 1 - 1 \\ &= 2 - 1 \\ &= 1 \end{aligned}$$

- c. From a rectangular solid of metal 42 cm by 30 cm by 20 cm, a cylindrical cavity of diameter 14 cm & depth 24 cm is drilled out. Find:
- i) Surface area of remaining solid
 - ii) Volume of remaining solid
 - iii) Weight of material drilled out if it weighs 7 gm per cm^3 [4]

Solution:

For rectangular solid(cuboid):

Length (l) = 42 cm

Width (b) = 30 cm

Height (h) = 20 cm

For conical cavity:

Diameter = 14 cm

$$\frac{14}{2}$$

Radius (r_c) = $\frac{14}{2} = 7$ cm

- i) Surface area of remaining solid

$$\begin{aligned} &= \text{TSA of cuboid} + \text{CSA of cylinder} - \text{Area of base of cylinder} \\ &= 2[lb + bh + lh] + 2\pi rh - \pi r_c^2 \\ &= 2[42 \times 30 + 30 \times 20 + 20 \times 42] + \pi r_c [2h - r_c] \\ &= 2[1260 + 600 + 840] + \frac{22}{7} \times 7[40 - 7] \end{aligned}$$

$$\begin{aligned}
&= 2[2700] + 22 \times 33 \\
&= 5400 + 726 \\
&= \underline{6126 \text{ cm}^2}
\end{aligned}$$

ii) Volume of remaining solid
= Volume of cuboid – Volume of cylinder
= $l \times b \times h - \pi r^2 h$

$$= (42 \times 30 \times 20) - \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24$$

= 25200 – 3696
= 21504 cm³

iii) Weight of material drilled out = Weight of cone
= $\frac{6126}{3} \times 7$
= 42882 gm
∴ Weight of material drilled out = 42.882kg

Question 3

- a. From the following data ,
25,10,40,88,45,60,77,36,18,95,56,65,7,0,38 and 83.find :
i Median
ii Upper quartile
iii Inter-quartile range

[3]

Solution-

Arranging the data in ascending order, we get
0,7,10,18,25,36,38,40,45,56,60,65,77,83,88,95
Median= $\frac{40+45}{2}=42.5$
2-Upper quartile=
 $=\frac{3n}{4}$
 $=3 \times \frac{16}{4}$
=12 th term =65
3-Lower quartile=
 $=\frac{n}{4}$
 $=\frac{16}{4}$
=4 th term =18
Inter-quartile range= Upper quartile- Lower quartile=47

- b. The volume of a rectangular solid is 3600 cm³. If it is 20 cm long and 9 cm high, find its total surface area.

[3]

Ans. breadth = $\frac{3600}{180} = 20 \text{ cm}$
TSA = $2[20 \times 9 + 9 \times 20 + 20 \times 20]$
= $2(180 + 180 + 400)$
= 2×760
= 1520 CM^2

c. Construct a triangle XYZ with angle Y equals to 75 and XY = 5.5 cm and YZ = 6.5 cm. and draw in circle in it. [3]

Ans. construction of triangle 1m
 Construction of 2 angle bisector 1m
 Drawing in circle 1m

Question 4

a. The mean of the numbers 6,x,7,y and 14 is 8. Express y in terms of x . [3]

Solution- Numbers of terms= 5

Mean= 8

Sum of numbers= 5 X 8 = 40-----1

As sum of numbers = 6+x+7+y+14-----2

From 1 and 2

b. A rectangular container whose base is square of side 6 cm holds water up to 3 cm from the top. When a solid cube is placed in the water and is completely submerged, the water rises to the top and 17 cm³ of water overflows. Find the edge of the cube.

[3]

Ans volume of cube = volume of empty space in cuboid + 17 cm³
 $Side^3 = l \times b \times h + 17 \text{ cm}^3$
 $= 6 \times 6 \times 3 + 17 \text{ cm}^3$
 $= 117 + 17 \text{ cm}^3$
 $Side^3 = 125$
 Side = 5 cm

c. The marks of 200 students in a test were recorded as follows:

| Marks % | No. of students |
|---------|-----------------|
| 10-19 | 7 |
| 20-29 | 11 |
| 30-39 | 20 |
| 40-49 | 46 |
| 50-59 | 57 |
| 60-69 | 37 |
| 70-79 | 15 |
| 80-89 | 7 |

Construct the frequency polygon of above frequency distribution. [4]

Question 5

a. Construct a triangle ABC with angle B equals to 90° and AC = 6.5 cm and BC = 6 cm. and draw circum circle on it. [3]

Ans. construction of triangle 1m
 Construction of side bisector 1m
 Drawing circle 1m

b. $1-2[(\tan 35/\cot 55)]^2 + [(\cot 55/\tan 35)]^2 - 3[(\sec 30/\operatorname{cosec} 60)]$ [3]

Ans

$$2[(\cot 55/\cot 55)]^2 + [(\tan 35/\tan 35)]^2 - 3[(\operatorname{cosec} 30/\operatorname{cosec} 60)] \\ 2 + 1 - 3 \\ = 0$$

c. In each case given below find the value of $\angle A$, where $0 \leq A \leq 90$

i. $\sin(90-3A)\operatorname{cosec} 42=1$

ii. $\cos(90-A)\sec 77=1$

[4]

Solution:

i. $\sin(90-3A)\operatorname{cosec} 42=1$

L.H.S-

$$\sin(90-3A) \times 1/\sin 42=1$$

$$\sin(90-3A)/\sin 42=1$$

$$90-3A/42=1$$

$$90-3A=42$$

$$-3A=-48$$

$$A=16$$

ii. $\cos(90-A)\sec 77=1$

$$\cos(90-A) \times 1/\cos 77=1$$

$$\cos(90-A)/\cos 77=1$$

$$(90-A)/77=1$$

$$(90-A)=77$$

$$-A=-13$$

$$A=13$$

Question 6

a. Draw an inscribing circle of a regular hexagon of side 5.8 cm and measure its radius. [3]

Ans. 4 angles of 120° 1m

Any two angle bisector 1m

In circle and radius 1m

b. If $2 \sin A - 1 = 0$. Show that: $\sin 3A = 3 \sin A - 4 \sin^3 A$. [3]

Solution:

$$2 \sin A - 1 = 0$$

$$2 \sin A = 1$$

$$\therefore \sin A = \frac{1}{2}$$

$$\text{But } \sin 30^\circ = \frac{1}{2}$$

$$\therefore \sin A = \sin 30^\circ$$

$$\therefore A = 30^\circ$$

$$\text{Now, L.H.S.} = \sin 3A \\ = \sin 3(30) \\ = \sin 90^\circ$$

$$\text{L.H.S.} = 1 \quad \dots (1)$$

$$\begin{aligned}
\text{R.H.S.} &= 3 \sin A - 4 \sin^3 A \\
&= 3 \sin 30^\circ - 4 \sin^3 30^\circ \\
&= 3 \left(\frac{1}{2}\right) - 4 \left(\frac{1}{2}\right)^3 \\
&= \frac{3}{2} - 4 \times \frac{1}{8} \\
&= \frac{3}{2} - \frac{1}{2} \\
&= \frac{3-1}{2} \\
&= \frac{2}{2}
\end{aligned}$$

$$\text{R.H.S.} = 1 \quad \dots (2)$$

from (1) & (2)

\therefore L.H.S. = R.H.S. ... hence proved.

- c. The height to the nearest cm, of 30 men are given below:
159,170,174,173,175,160,161,164,163,165,164,171,162,170,177,185,181,180,175,165,186
,174,168,168,176,176,165,175,167,180
Using class intervals 155-160,160-165,-----draw up a grouped frequency distribution table[4]

Ans.

| Height (in cm) | Tally mark | No. of person |
|------------------|------------|---------------|
| 155-160 | | 1 |
| 160-165 | | 6 |
| 165-170 | | 6 |
| 170-175 | | 6 |
| 175-180 | | 6 |
| 180-185 | | 3 |
| 185-190 | | 2 |
| total | | 30 |

Question 7

- a. Evaluate: $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \operatorname{cosec} 31^\circ$.

[3]

Solution:

$$3 \cos 80^\circ \cdot \operatorname{cosec} 10^\circ + 2 \cos 59^\circ \cdot \operatorname{cosec} 31^\circ$$

$$\begin{aligned}
&= 3 \sin (90 - 80)^\circ \cdot \operatorname{cosec} 10^\circ + 2 \sin(90 - 59)^\circ \operatorname{cosec} 31^\circ \\
&= 3 \sin 10^\circ \cdot \operatorname{cosec} 10^\circ + 2 \sin 31^\circ \cdot \operatorname{cosec} 31^\circ \\
&= 3 \times 1 + 2 \times 1 \qquad \dots \sin \theta \cdot \operatorname{cosec} \theta = 1 \\
&= 3 + 2 \\
&= 5
\end{aligned}$$

b. The marks of 20 students in a test were as follows:

2, 6, 8, 9, 10, 11, 11, 12, 13, 13, 14, 14, 15, 15, 15, 16, 16, 18, 19 & 20.

Calculate: (i) the mean, (ii) the median, (iii) the mode

[3]

Solution:

$$\text{i) } \sum x = 2 + 6 + 8 + 9 + 10 + 11 + 11 + 12 + 13 + 13 + 14 + 14 + 15 + 15 + 15 + 16 + 16 + 18 + 19 + 20.$$

$$\sum x = 257$$

$$n = 20$$

$$\text{Mean} = \frac{\sum x}{n}$$

$$= \frac{257}{20}$$

$$= 12.85$$

$$\text{Mean} = 12.85$$

$$\text{ii) } N = 20 \text{ (even)}$$

$$\left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ term}$$

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{ term} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ term}}{2}$$

$$= \frac{\left(\frac{20}{2}\right)^{\text{th}} \text{ term} + \left(\frac{20}{2} + 1\right)^{\text{th}} \text{ term}}{2}$$

$$= \frac{10^{\text{th}} \text{ term} + 11^{\text{th}} \text{ term}}{2}$$

$$= \frac{13 + 14}{2}$$

$$= \frac{27}{2}$$

$$= 13.5$$

$$= 13.5$$

$$\text{Median} = 13.5$$

iii) No. with highest frequency = 15

∴ Mode = 15

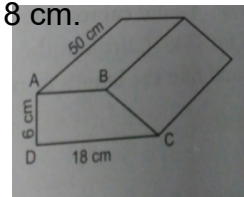
c. The diagram represents a solid block of wood of length 50 cm. the cross section is trapezium ABCD with AB = 10 cm, AD = 6 cm and CD = 18 cm.

Find: i. its volume ii total surface area

Ans. Area of cross section = $\frac{1}{2} (10 + 18) 6 = 84 \text{ cm}^2$

Volume of solid = $84 \times 50 = 4200 \text{ cm}^3$

Perimeter of cross section = $10 + 6 + 18 + 10 = 44 \text{ cm}$



[4]

$$\begin{aligned}
\text{TSA} &= 44 \times 50 + 2 (\text{area of trapezium}) \\
&= 2200 + 2 \times 84 \\
&= 2200 + 168 \\
&= 2368 \text{ cm}^2
\end{aligned}$$

Question 8

- a. A cylinder with radius 333 units and its volume is 868686 cubic units. Find the height of the cylinder. [3]

$$\text{Ans. } 868686 = \frac{22}{7} \times 333 \times 333 \times h$$

$$h = \frac{868686 \times 7}{22 \times 333 \times 333}$$

$$h = \frac{6080802}{2439558} = 2.493$$

- b. Prove that $\frac{\cos A}{1 + \sin A} + \tan A = \sec A$ [3]

$$\text{LHS} = \frac{\cos A}{1 + \sin A} + \tan A$$

$$= \frac{\cos A}{1 + \sin A} + \frac{\sin A}{\cos A} \quad [1]$$

$$= \frac{\cos^2 A + \sin A + \sin^2 A}{(1 + \sin A)\cos A}$$

$$= \frac{1 + \sin A}{(1 + \sin A)\cos A} \quad (\because \sin^2 A + \cos^2 A = 1) \quad [1]$$

$$= \frac{1}{\cos A}$$

$$= \sec A$$

- c. Draw a histogram and hence estimate the mode for the following frequency distribution: [4]

| Class | Frequency |
|-------|-----------|
| 0-10 | 2 |
| 10-20 | 8 |
| 20-30 | 10 |
| 30-40 | 5 |
| 40-50 | 4 |
| 50- | 3 |

Solution - Value of mode = 23

