GREENLAWNS SCHOOL, WORLI
Terminal Examination 2017
MATHEMATICS

STD: IX Marks: 80
Date: 26/09/2017 Time: 2½hrs

Section A
(Attempt **all** the questions from this section)

Question 1

a. For the following set of data, find median:
10, 75, 3, 81, 17, 27,4, 48, 12, 47, 9 & 1 **[3]**

b. construct a regular hexagon of side 4 cm. **[3]**

c. In Δ ABC angle A = 60̊ and angle C = 40̊ and bisector of angle ABC meet AC at point p. Show that BP = CP **[4]**

Question 2

a. If $a^x = b$, $b^y = c$ and $c^z = a$, prove that : $xyz = 1$. **[3]**

b. A sum of ` 9600 is invested for 3 years at 10% p.a. CI
i) What is sum due at the end of first year?
ii) What is sum due at the end of second year?
iii) Hence write down the CI for 3rd year. **[3]**

c. Solve for x:
(i) $\log (x + 5) + \log (x – 5) = 4 \log 2 + 2 \log 3$. (ii) $\log (x + 3) – \log (x – 3) = 1$ **[4]**

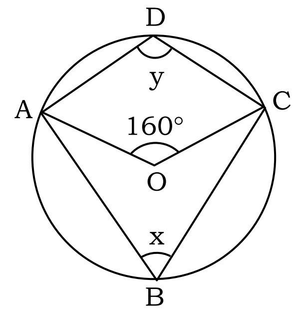
Question 3

a. The difference of the squares of two consecutive even natural numbers is 92. Taking x as the smaller of the two numbers, form an equation in x and hence find the larger of the two numbers. **[3]**

b. If $\log_5 x = y$, find $5^{2y + 3}$ in terms of x. **[3]**

c. A chord CD of a circle, whose centre is O, is bisected at P, by diameter AB. Given: OA = OB = 15cm, OP = 9cm
Calculate lengths of i) CD ii) AD iii) CB **[4]**

Question 4



a. In the figure, O is centre of the circle $\& \angle AOC = 160^\circ$. Prove: $3\angle y - 2\angle x = 140^\circ$ **[3]**

b. Factorize: $4(2x - 3y)^2 - 8x + 12y - 3$ **[3]**

c. Simplify

(i) $\dfrac{8^{3a} \times 2^5 \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}}$ (ii) $\dfrac{3 \times 27^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 27^n}$ **[4]**

Section B

(Attempt any **four** questions from this section)

Question 5

a. On the side AB and AC of a triangle ABC, equilateral triangle ABC and ACE are drawn.
Prove that: i. angle CAD = angle BAE ii. CD = BE **[3]**

b. Two angles of eight sided polygon are 142° and 176°**.** If the remaining angles are equal to each other; find the magnitude of each of the equal angles. **[3]**

c. Draw a histogram & hence estimate the mode for following frequency distribution: **[4]**

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	2	8	10	5	4	3

Question 6

a. In a triangle ABC ; AB = AC .Show that the altitude AD is median also. **[3]**

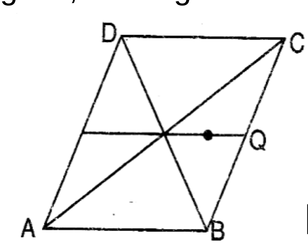
b. Solve: $\frac{3}{x} - \frac{2}{y} = 0$ and $\frac{2}{x} + \frac{5}{y} = 19$. Hence, find 'a' if $y = ax + 3$ **[3]**

c. The diagonals of a parallelogram ABCD intersect at point O. Through O, a straight line is drawn parallel to AB which meets AD in P and BC in Q.

Prove that:

(i) P and Q are mid-points of AD and BC respectively.

(ii) Area of $\Delta OAB = \frac{1}{4}$ times the area of parallelogram ABCD. **[4]**



Question 7

a. If x = 2 √3 + 2 √2, find:

(i) $\frac{1}{x}$ (ii) $x + \frac{1}{x}$ (iii) $\left(x + \frac{1}{x}\right)^2$ **[3]**

b. Marks obtained by 9 students are given below:

60, 67, 52, 76, 50, 51, 74, 45 & 56

i) Find arithmetic mean

ii) If marks of each student be increased by 4; what will be new arithmetic mean. **[3]**

c. Solve: $\frac{2x+1}{10} - \frac{3-2x}{15} = \frac{x-2}{6}$, Hence, find y, if $\frac{1}{x} + \frac{1}{y} + 1 = 0$. **[4]**

Question 8

a. If m = log 20 and n = log 25, find the value of x, so that : 2 log (x – 4) = 2 m – n. **[3]**

b. Construct a rhombus ABCD when one of its side is 6cm and angle A is 60° **[3]**

c. The sum of the digits of a two digit number is 7. If the digits are reversed, the new number decreased by 2, equals twice the original number. Find the number. **[4]**

Question 9

a. The sum of two numbers is 9 and product is 20 find the sum of their

i. squares ii. Cubes. **[3]**

b. It is estimated that every year the value of a machine depreciates by 20% of its value at the beginning of the year. Calculate the original value of machine, if its value after two years is Rs.10240. **[3]**

c. In Δ ABC, D is a point of AB, such that AD = $\frac{1}{4}$ AB and E is a point on AC such that AE = $\frac{1}{4}$ AC.

Prove that: DE = $\frac{1}{4}$ BC. **[4]**

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Answer key
Section A
(Attempt **all** the questions from this section)

Question 1

a. For the following set of data, find median:
10, 75, 3, 81, 17, 27,4, 48, 12, 47, 9 & 1 **[3]**

Solution:
Asc. Order:

3, 4, 9, 10, 12, 15, 17, 27, 47, 48, 75, 81

$$n = 12 \text{ (even)}$$

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{term} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{term}}{2}$$

$$= \frac{\left(\frac{12}{2}\right)^{\text{th}} \text{term} + \left(\frac{12}{2}+1\right)^{\text{th}} \text{term}}{2}$$

$$= \frac{6^{\text{th}} \text{term} + 7^{\text{th}} \text{term}}{2}$$

$$= \frac{15+17}{2}$$

$$= \frac{32}{2}$$

Median = <u>16</u>

b. construct a regular hexagon of side 4 cm. **[3]**

c. In Δ ABC angle A = 60° and angle C = 40° and bisector of angle ABC meet AC at point p. Show that BP = CP **[4]**

Question 2

a. If $a^x = b$, $b^y = c$ and $c^z = a$, prove that : $xyz = 1$. **[3]**

b. A sum of ` 9600 is invested for 3 years at 10% p.a. CI
- i) What is sum due at the end of first year?
- ii) What is sum due at the end of second year?
- iii) Hence write down the CI for 3rd year. **[3]**

Solution:

For 1st year:

P = `9600

R = 10% p.a.

T = 1 year

$$I = \frac{P \times R \times T}{100}$$

$$= \frac{9600 \times 10 \times 1}{100}$$

I = `960

A = P + I

= 9600 + 960 = `10560

For 2nd year:

P = `10560

R = 10% p.a.

T = 1 year

$$I = \frac{P \times R \times T}{100}$$

$$I = \frac{10560 \times 10 \times 1}{100} = \text{` } 1056$$

A = P + I

= 10560 + 1056

= ` 11616

i) Sum due after 1 year = ` 1056

ii) Sum due after 2nd year = ` 11616

iii) Req. Diff. = 11616 – 10560

= ` 1056 (i.e. I_2)

iv) Interest on difference for 1 year

$$I = \frac{1056 \times 10 \times 1}{100} = \text{` } 105.60$$

v) $$\begin{pmatrix}\text{CI for} \\ 3^{\text{rd}} \text{ year}\end{pmatrix} = \begin{pmatrix}\text{CI for} \\ 2^{\text{nd}} \text{ year}\end{pmatrix} + \begin{pmatrix}\text{Int on it} \\ \text{for 1 year}\end{pmatrix}$$

= 1056 + 105.60

CI for 3rd year = ` 1161.60

c. Solve for x:
(i) $\log (x + 5) + \log (x - 5) = 4 \log 2 + 2 \log 3$. (ii) $\log (x + 3) - \log (x - 3) = 1$ **[4]**

Question 3

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c. A chord CD of a circle whose centre is O, is bisected at P, is bisected at P, by diameter AB.
Given: OA = OB = 15cm, OP = 9cm
Calculate lengths of i) CD ii) AD iii) CB **[4]**

Solution:

Statement **Reason**

1) CP = PD given

2) OP ⊥ CD line joining centre& mid pt of chord is $\perp^r$ to chord.

3) OA = OB = OC = 15cm Radii of same ⊙

4) In rt△ OPC,

$(OC)^2 = (OP)^2 + (CP)^2$ Pythagoras theorem

5) $(Cp)^2 = (OC)^2 + (OP)^2$

$= (15)^2 + (9)^2$ substitution

$(CP)^2 = 144$ Taking sq. root on both sides from (1) & given

$\therefore CP = 12cm$

6) $CD = 2 \times CP = 2 \times 12$

CD = 24 cm

7) AP = AO + OP,AP = 24cm

8) In rtΔ APD,

$(AD)^2 = (AP)^2 + (PD)^2$ Pythagoras theoremFrom (1) (5) & (7)

$= (24)^2 + (12)^2$

$= 576 + 144 (AD)^2 = 720$ Taking Sq root on both sides

$\boxed{AD = 26.83cm}$

9) PB = OB – OP = 15 – 9 PB = 6cm (O – P – B)

10) In rtΔBPC,

$(BC)^2 = (CP)^2 + (PB)^2 = (12)^2 + (6)^2$ Pythagoras theorem from 5 & 9

$= 144 + 36$

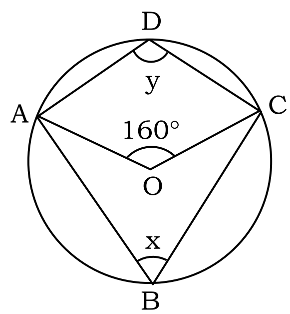
$(BC)^2 = 180$

$\therefore BC = 13.416$ Taking sq. root on both sides

$\boxed{BC = 13.42cm}$

Question 4

a. In the figure, O is centre of the circle &$\angle AOC = 160^\circ$. Prove: $3\angle y - 2\angle x = 140^\circ$ **[3]**



Solution:

Statement	**Reason**
1) $\angle AOC = 2\angle ABC$	Central angle is twice the angle at remaining circumference
2) $160^\circ = 2x$	Substitution
	$\therefore \boxed{x = 80^\circ}$
3) $\angle ABC + \angle ADC = 180^\circ$	Opp angles of cyclic □ are supplementary
4) $x + y = 180^\circ$	substitution
	$y = 180^\circ - x^\circ$
$y = 180^\circ - 80^\circ$	from (2)
	$\boxed{y = 100^\circ}$
5) $3\angle y - 2\angle x = 3(100) - 2(80) \rightarrow$	from 2 & 4
	$= 300^\circ - 160^\circ$
	$3\angle y - 2\angle x = 140^\circ$

hence proved.

b. Factirise : $4(2x - 3y)^2 - 8x + 12y - 3$ [3]

c. Simplify :

(i) $\dfrac{8^{3a} \times 2^{5} \times 2^{2a}}{4 \times 2^{11a} \times 2^{-2a}}$ (ii) $\dfrac{3 \times 27^{n+1} + 9 \times 3^{3n-1}}{8 \times 3^{3n} - 5 \times 27^{n}}$ [4]

Section B

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Question 5

a. On the side AB and AC of a triangle ABC, equilateral triangle ABC and ACE are drawn.
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b. Two angles of eight sided polygon are 142° and 176°**.** If the remaining angles are equal to each other; find the magnitude of each of the equal angles. [3]

c. Draw a histogram & hence estimate the mode for following frequency distribution:

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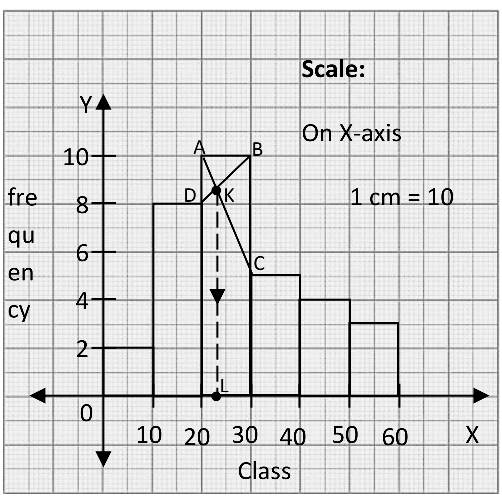
Frequency	2	8	10	5	4	3

Solution:



From graph: Join AC & BD } Join AC & BD

Mode = <u>23</u>

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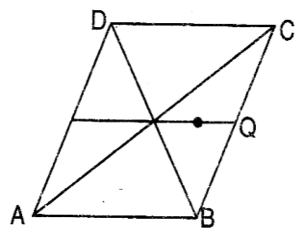

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a. If $x = 2\sqrt{3} + 2\sqrt{2}$, find :

(i) $\frac{1}{x}$ (ii) $x + \frac{1}{x}$ (iii) $\left(x + \frac{1}{x}\right)^2$ **[3]**

b. Marks obtained by 9 students are given below:
60, 67, 52, 76, 50, 51, 74, 45 & 56
i) Find arithmetic mean
ii) If marks of each student be increased by 4; what will be new value of arithmetic mean. **[3]**

Solution:

i) Arithmetic mean $= \frac{\sum x}{n}$

$\sum x = 60 + 67 + 52 + 76 + 50 + 51 + 74 + 45 + 56$

$\therefore \sum x = 531$

$n = 9$

Arithmetic mean $= \frac{\sum x}{n}$

$= \frac{531}{9}$

$\therefore$ Arithmetic mean = 59

ii) When marks of each student is increased by 4,

New Arithmetic Mean = Original Mean + 4

= 59 + 4

= 63

c. Solve :$\frac{2x+1}{10} - \frac{3-2x}{15} = \frac{x-2}{6}$, Hence, find y, if $\frac{1}{x}+\frac{1}{y}+1=0$. **[4]**

Question 8

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Question 9

a. The sum of two numbers is 9 and product is 20 find the sum of their
i. squares ii. Cubes. **[3]**

b. 8.It is estimated that every year the value of a machine depreciates by 20% of its value at the beginning of the year. Calculate the original value of machine, if its value after two years is Rs.10240. **[3]**

Solution:

For 1st year:

Principal = ` P … (Assumption)

Rate = 20%

T = 1 year

$I = \frac{P \times R \times T}{100}$

$I = \frac{P \times 20 \times 1}{100} = ` \frac{2P}{10}$

A = P + I … (∵ depreciation)

$= \frac{P}{1} - \frac{2P}{10}$

$A = \frac{10P - 2P}{1} = ` \frac{8P}{10}$

For 2nd year:

$P = ` \frac{8P}{10}$

R = 20% p.a.

T = 1 year

$I = \frac{P \times R \times T}{100}$

$$I = \frac{8P \times 20 \times 1}{10 \times 100}$$

$I =$ ` $\frac{16P}{100}$

$A = P - I$ … ($\because$ depreciation)

$$= \frac{8P}{10} - \frac{16P}{100}$$

$$A = \frac{80P - 16P}{100}$$

$$\therefore 10240 = \frac{64P}{100}$$

$$\therefore P = \frac{10240 \times 100}{64}$$

$P =$ ` 16000

Ans.: The original value of machine
= ` 16000.

c. In Δ ABC, D is a point of AB, such that AD = $\frac{1}{4}$ AB and E is a point on AC such that AE = $\frac{1}{4}$ AC. Prove that: DE = $\frac{1}{4}$ BC. **[4]**